

Conditional CAPM model with a logit variable

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ABSTRACT: Despite the criticisms, the improvement of the static CAPM, giving rise to new conditional models, brings greater security to the investor throughout the business cycle. The CAPM and its static versions were and are of great importance in finance. Nowadays we find more complex adaptations of the CAPM model, which allow us to have answers to questions in finance that for a long time remained unsolved. Given this panorama and considering all this great discussion about the validity of the CAPM, this article seeks to present the conditional model. To achieve such an objective, the tests of the conditional models (beta varying over time) will be studied, using Artificial Intelligence. These tests are convenient for incorporating variances and covariances that change over time. Among the tests of the conditional models, we highlight that of Jagannathan and Wang (1996) with the application of a Logit variable.

Keywords: Finance; CAPM; conditional model, Logit variable

I. INTRODUCTION

This article presents five sections that discuss the following aspects: in section 1 the introduction of the work. Section 2 shows the general aspects of the static CAPM of Sharpe, Lintner, Black, Mossin and Treynor. Section 3 presents the Conditional CAPM model. Section 4 the econometric tests with the introduction of analysis through AI(Logit) While section 5 presents some additional information. And, finally, section 6 presents the conclusions and recommendations. In 1952, Harry Markowitz published the famous work "Portfolio Selection" which

stirred the academic environment of the time. Markowitz shows that portfolio analysis begins with the analysis of individual securities and ends with obtaining the efficient portfolio that best meets the investor's objectives. Starting from this point, the efficient frontier of portfolios is built and, among these, the one that best satisfies the investor in relation to the expected return and risk is selected.

Based on the ideas exposed by Markowitz, William Sharpe developed the "Single Index Model", in 1963, which sought to simplify the variance matrix of the Markowitz model. In 1964 Sharpe published his famous article "Capital Asset Prices: A Theory of Market Equilibrium under conditions of risk" thus structuring the "Capital Asset Pricing Model", a model for pricing assets in risky securities markets in equilibrium, which is widely used and has important applications.

The CAPM is defined as a model that relates the expected profitability of an asset or good, in a market in equilibrium, with its non-diversifiable risk, also known by the name of beta. Independent of Sharpe, other authors also formulate the

CAPM, in its static version. Among them Lintner (1965), Mossin (1966) and Treynor. This version of the static or unconditional CAPM has consistent results when performing empirical tests to verify the model's ability to adhere to the real economy.

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In all tests of the unconditional CAPM, including that of Fama and MacBeth (1974), Black, Jensen and Scholes (1972) it was assumed that the beta would be static, that is, that the risk systematic of the assets would not change.

Haugen (1986) shows that Black, Jensen and Scholes considered that there should exist a positive linear relationship between the betas and the expected return rates. As a consequence of this fact, Black, Jensen and Scholes (1972) find in their CAPM test a relationship positive between profitability and beta.

In the studies by Fama and MacBeth (1974), the betas of a period are used to predict the returns in a later period. Fama and MacBeth (1974) show that in these studies empirical tests of the CAPM assume that the betas remain constant over time and the return of portfolios containing stocks traded in the markets is a good proxy for the return expected from the market.

Merton (1973) shows, in his general multi-beta equilibrium model, that the return conditional expected is linear with the conditional betas, including the market beta.

Merton (1973) developed the Consumption Capital Asset Pricing Model (ICAPM). This model aimed to generalize Sharpe's (1964) CAPM model to an intertemporal context. The original ICAPM starts from the hypothesis that investors consumed all their wealth after one period, so that wealth and consumption ended up being confused.

Meanwhile, the growth in the number of empirical studies examining the productive capacity of the static version of the CAPM has become evident over the last two decades. The results obtained in these studies show that the static CAPM is unable to reasonably explain the cross-sectional variation of the average return of the analyzed portfolios.

Recent literature has focused on studies of CAPM models with time-varying beta. These models are known as conditional versions of the CAPM. This conditional model is a convenient way to incorporate variances and covariances that change over time, which we empirically verify in financial time series. Next, we verify the conditional CAPM represented in the equation below:

$$E[R_{it} | I_{t-1}] = \gamma_{t-1} + \beta_{it-1} \gamma_{1t-1} \quad (1.0)$$

Where I_{t-1} is the information set of investors at the end of period $t-1$ and β_{it-1} is the conditional beta of asset i in period $t-1$ defined as:

$$\beta_{it-1} = \text{Cov}(R_{it}, R_{mt} | I_{t-1}) / \text{Var}(R_{mt} | I_{t-1}) \quad (1.1)$$

γ_{0t-1} is the conditional expected return on the portfolio with beta equal to zero, γ_{1t-1} is the conditional market risk premium. Jagannathan and Wang (1996) use this equation to explain the cross-sectional variation of the conditional expected return on different assets.

However, analyzing the static CAPM pragmatically, most articles

agree that the model is not "literally" true. Confirming the idea above, Roll (1977) reports an observation in which the market portfolio is not observable. It also shows the possibility that the value-weighted index is a poor proxy for the aggregate wealth of the portfolio, thus the reason for the unsatisfactory performance of CAPM in empirical studies.

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Stambaugh (1982) also shows that tests on CAPM are not very sensitive in relation to the approximation made for the market portfolio.

The model specification is obtained in a static scenario and would only have intertemporal value by admitting strong assumptions. However, as no theory exactly represents reality, CAPM continues to be accepted as a benchmark when validated empirically.

II. GENERAL ASPECTS OF THE STATIC CAPM OF SHARPE, LINTNER, BLACK, MOSSIN AND TREYNOR

The static CAPM model of Sharpe, Lintner, Black, Mossin and Treynor shows that R_i denotes the return of asset i and R_m the return of the market portfolio of all tradable assets in the economy. Black's (1972) version of CAPM dispenses with a risk-free rate and assumes that the lending rate is different from the borrowing rate. The expected value of the return of the asset that has zero covariance with the market portfolio is treated as unobservable and will be one of the parameters to be estimated. Black's model is defined as:

$$E(R_i) = E(R_z) + [E(R_m) - E(R_z)]\beta_i \quad (2.0).$$

where: $E(R_z)$ is the return of the zero-beta asset.

Fama and French (1992) empirically examined the version of CAPM described above and found the estimated value of γ_1 close to zero. They also identified the weak relationship between the average return and beta as strong evidence against CAPM. As seen previously, one of the CAPM hypotheses is that it assumes the analysis of a single period. However, the real world is dynamic and expected returns and betas vary over time. When expected returns and betas do not vary over many periods the relationship between the expected return and the beta will likely be weak (static CAPM).

Jagannathan and Wang (1996), suggested that in a hypothetical economy econometricians consider only two stocks where there are only two maturity dates.

The betas of the first stock are 0.5 and 1.25 (0.875 equivalent to the average of the betas). Similarly for the first and second dates, respectively. The betas corresponding to the second stock are 1.5 and 0.75 (1.125 equivalent to the average of the betas). The authors also consider that the expected

risk premium is 10% on the first maturity date and 20% on the second. So if the CAPM is

valid in each period, the expected risk premium of the first stock will be 5% on the first maturity date and 25% on the second. The expected risk premium on the second stock will be 15% on both maturity dates. Therefore, econometricians who ignore the fact that betas and risk premiums vary over time will erroneously conclude that the CAPM is not satisfactory, since the two stocks earn a risk premium of 15% and their average betas differ.

Several empirical studies on pricing models reported in the literature show

a significant statistical variation of betas over time. Mandelker (1974) examines in some empirical studies the reaction of prices in stock markets determined by historical events that affected the financial market and as a consequence the financial literature. Such studies were, later, important and allowed the analysis of beta variations over time.

According to Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), Shanken and Weinstein (1990), Hansen and Singleton (1982), Connor and Korajczyk (1988), empirical implementations of these static models have failed greatly in relation to explanatory power.

Fama and French (1993) through multivariate tests find two variables that explain most of the cross-section variations of average returns: the book-to-market value ratio has a positive relationship with stock returns while the size variable has a quite significant negative relationship. These results suggest that risk has multidimensional characteristics.

Tests of the static version of the CAPM seen in Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), assume that expected returns are constant, that the market portfolio is observable and that asset betas are stationary in fixed periods. Cross-sectional tests are performed through regressions of unconditional expected returns and also unconditional betas. Through these static CAPM tests it was possible to observe that when the expected returns are constant the relationship between beta and return does not satisfactorily explain the average cross-sectional variation of returns. On the other hand, asset pricing models at a conditional level (conditional CAPM), allow expected returns to vary over time. However, all these studies assume that conditional covariances are constant.

III. THE CONDITIONAL CAPM

Ribenboim (2002) reinforces the idea that the conditional CAPM allows the risk premium of an asset to change based on the variation of one or more of the following components: the conditional variance of the market portfolio return, the conditional covariance between the asset return and the market portfolio return and the market portfolio risk premium. When the conditional covariance between each asset and the market is proportional to the conditional variance, we fall back into the unconditional CAPM.

Jagannathan and Wang (1996) in their conditional CAPM use the subscript t to indicate the relevant time period. R_{it} denotes the return of asset i in period t and R_{mt} the return of the aggregate wealth of the portfolio of all assets in the economy in period t .

$$E[R_{it} | I_{t-1}] = \gamma_0 + \gamma_1 \beta_{it-1} + \gamma_2 \epsilon_{it-1} \quad (3.1)$$

Where I_{t-1} is the information level of investors at the end of period $t-1$ and β_{it-1} is the conditional beta of asset i in period $t-1$ defined as:

$$\beta_{it-1} = \text{Cov}(R_{it}, R_{mt} | I_{t-1}) / \text{Var}(R_{mt} | I_{t-1}) \quad (3.2)$$

γ_0 is the conditional expected return on the portfolio with beta equal to zero, γ_1 is the conditional market risk premium. Jagannathan and Wang (1996) to explain the cross-sectional variation of the unconditional expected return on different assets, place the unconditional expectation on both sides of equation (3.3), finding the equation below:

$$E[E[R_{it}]] = \gamma_0 + \gamma_1 E[\beta_{it-1}] + \gamma_2 E[\epsilon_{it-1}] \quad (3.3)$$

$$\gamma_0 = E[R_{it} | \beta_{it-1} = 0] \quad \gamma_1 = E[\beta_{it-1}]$$

$$\text{where: } \gamma_1 = E[\beta_{it-1}] = E[\beta_{it-1}]$$

Here, γ^1 is the expected market risk premium, and β_i is the expected beta. If the covariance between the conditional beta β_{it-1} of assets i and the conditional market risk premium is zero or a linear function of the expected beta for an arbitrary choice of assets i , equation (3.3) will resemble the static CAPM, where the expected return is a linear function of the expected beta. In any case, generally, the conditional market risk premium and the conditional betas are correlated.

However, according to the works of Keim and Stambaugh (1986), Breem, Glosten, and Jagannathan (1989), Fama and French (1989), Chen (1991), and Ferson and Harvey (1991) show that the expected risk premium of assets and the conditional betas are not constant. Therefore, generally the

last term of equation (3.3) is not zero and the unconditional expected return is not a function linear of the expected beta. This fact motivated Jagannathan and Wang (1996) to decompose the conditional beta of asset i into two orthogonal components to project the conditional beta of the market risk premium. For each asset i , the sensitivity of the market premium beta (denoted by θ) and the residual beta (denoted by η_{it-1})

$$\text{Where: } \theta_i = \text{Cov}(\beta_{it-1}, \gamma^1_{t-1}) / \text{Var}(\gamma^1_{t-1}) \quad (3.4)$$

$$\eta_{it-1} = \beta_{it-1} - \beta_i - \theta_i (\gamma^1_{t-1} - \gamma) \quad (3.5)$$

In the expression above, θ_i measures the sensitivity of the conditional beta to the risk premium of market. For each asset i we have

$$\beta_{it-1} = \beta_i + \theta_i (\gamma^1_{t-1} - \gamma) + \eta_{it-1} \quad (3.6)$$

$$E[\eta_{it-1}] = 0, \quad (3.7)$$

$$E[\eta_{it-1} \gamma_{it-1}] = 0$$

$$R_{it} = \gamma + \beta_i + \theta_i (\gamma^1_{t-1} - \gamma) + \eta_{it-1}$$

(3.8)

In equation (3.9) each conditional beta is decomposed into three orthogonal parts: The first part corresponds to the expected beta which is the constant; the second part corresponds to the random variable that is perfectly correlated with the market risk premium; the last part refers to the zero mean which is not correlated with the market risk premium. According to Jagannathan and Wang (1996), substituting equation (3.4) into (3.6), we have:

$$E[R_{it}] = \gamma + \beta_i + \text{Var}(\gamma^1_{t-1}) \theta_i \quad (3.9)$$

Therefore the cross-section of the unconditional expected return of assets i is a linear function of the expected beta and the beta premium sensitivity. The beta sensitivity of assets measures the instability of asset betas during the business

cycle.

Jagannathan and Wang (1996) verify that portfolios with high expected betas have high unconditional expected returns. Likewise, portfolios with betas that are prone to vary with the market risk premium and, therefore, are less stable during the business cycle, also have high unconditional expected returns.

The complete specification of the conditional CAPM test by Jagannathan and Wang (1996) is

verified in equation (3.9) and requires the estimation of the expected beta, β_i , and the beta sensitivity of the premium, θ_i , visualized in equation (3.9), as well as the other parameters.

The objective is to examine whether unconditional expected returns are consistent with the conditional CAPM. In equation (3.10) it can be seen that residual betas do not affect the unconditional expected return. Thus, two types of unconditional betas were defined:

$$\beta_i = \text{Cov}(R_{it}, Y_{it}) / \text{Var}(Y_{it}) \quad (3.10)$$

The first unconditional beta refers to the portfolio beta and the second to the premium beta. They measure the average portfolio risk and the market instability risk respectively.

It is verified in equation (3.10) that the unconditional expected return is a linear function of the two unconditional betas. This is verified in the following theorem:

Theorem 1. If β_i is not a linear function of α then there exist some constants α_0, α_1 , and α_2 such

that the equation,

$$E[R_{it}] = \alpha_0 + \alpha_1 \beta_i + \alpha_2 \theta_i \quad (3.11)$$

is valid for every asset i .

IV. ECONOMETRIC TESTS WITH THE LOGIT VARIABLE

Jagannathan and Wang (1996), define that the "LOGIT VARIABLE" serves as the basis for their empirical study. There are several ways to examine whether the data are consistent with the cited model. According to this model, the static expected return of the asset is only a linear function of the three betas. A specification test that could be performed would be to examine whether any other variable has the capacity to explain the cross-section of the

average return not explained by the three betas of the model. One can investigate, as Jagannathan and Wang did, the existence of the size effect in the residuals of the PL model. Market size is defined as the logarithm of the asset's market value. Thus $\log(ME_i)$ denotes the time series average for the size of asset i . The possibility of some size effect is examined by including $\log(ME_i)$ in the

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Logit model:

$$E[R_{it}] = c_0 + c_{size} \log(ME_i) + c_{vw} \beta_i^{vw} + c_{labor} \beta_i^{labor} + c_{prem} \beta_i^{prem} + c_{psi} \beta_i^{psi} \quad (4.0)$$

If the model with this variable is satisfactory, then the coefficient c_{size} must be zero and there will be no size effect on the residual. The unconditional or static model of equation (4.0) can be satisfactorily estimated by cross-sectional regression, which is the method proposed by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973). To compare the performance of different empirical specifications, the R^2 of the cross-sectional regression is used which shows the fraction of the variation in the average cross-sectional return that can be explained by the model. It is also examined whether, c_{vw} , c_{labor} , and c_{prem} are different from zero after error estimation.

An evaluation of these specifications can be made using the generalized method of moments. Following this purpose, some restrictions imposed on the PL model must be considered. According to Dybvig and Ingersoll (1982) the definitions of

β_i^{vw} , β_i^{labor} , and β_i^{prem} can be substituted into the PL model and, by rearranging the variables, the following equation is obtained:

$$E[R_{it}] = c_0 + c_{size} \log(ME_i) + c_{vw} \beta_i^{vw} + c_{labor} \beta_i^{labor} + c_{prem} \beta_i^{prem} + c_{psi} \beta_i^{psi} + \delta_{psi,t} \quad (4.1)$$

4.1)

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Where α_0 , α_{vw} , α_{prem} and δ_{labor} are model constants.

According to Jagannathan and Wang (1996) and Ross (1976), as long as the financial market satisfies the law of one price, there will exist at least some variables d_t , such that:

$$E[R_{it} d_t] = 1, \quad (4.2)$$

Where d_t is called the stochastic discount factor and is given by:

$$d_t = \alpha_0 + \alpha_{vw} R_{vw} + \alpha_{prem} R_{prem} + \delta_{labor} R_{labor} + \psi_t R_{labor} \quad \text{Logit}$$

(4.3)

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that depends on the parameters $\beta \equiv (\beta_0, \beta_{vw}, \beta_{\text{prem}}, \text{Logit})$).

Assuming there are N assets used in the econometric test and with the presence of an N -dimensional vector it is correct to specify, in the LOGIT model, the N -dimensional pricing errors, $E[w_t(\beta)]$, which should be zero.

V. ADDITIONAL INFORMATION

The conditional model developed by Jagannathan and Wang (1996) resembles the multi-factor model developed by Chen, Roll, and Ross (1986). The natural question that arises is whether the premium lag factor and the human capital growth factor used in the specifications of the Jagannathan and Wang (1996) model are satisfactory proxies for macroeconomic factors identified by Chen, Roll, and Ross (1986).

Jagannathan and Wang (1998) show that market characteristics are generally used to detect misspecification of errors. If the linear beta pricing model is correct, other variables associated with market characteristics added to the model should not explain the cross-sectional variation of expected return. In this case, the t -statistic values of these variables should be insignificant. Therefore, significant t -values are seen as a rejection factor of the PSI model. Commonly, market size is used as a characteristic, which is defined as the log of market capitalization. When the linear beta pricing model is satisfactory, the t -values converge to a normal distribution with finite variance. In any case, when factors or models are misspecified,

it is observed that the t -values, associated with market characteristics, generally converge to infinite probabilities.

Banz and Rolf (1981) were the first to use market size to

examine the CAPM. They show in their study that the size effect is not linear with the market proportion (or the log of the market proportion). The effect is not stable over time. The size effect exists, but the reason for its existence is not clearly explained. However, the doubt remains whether market size can be viewed as a satisfactory proxy.

For Basu and Sanjoy (1983), the relationship between excess return and market size can be considered linear in relation to the log of market capitalization size. The transformation is important because the kurtosis of the market size distribution shows that the excess returns attributed to size are not constant over time. It also shows that different estimation methodologies can generate different conclusions about the size effect. In this sense, the introduction of the analysis of artificial intelligence can reduce the differences regarding the size effect. Chan, Chen and Hsieh (1985) find that high average returns in markets small are offset by the high risk which is more significant, since, in this analysis, the risk is the covariance of the portfolio return with the risk premium. The inability of the market beta to capture these risks leads to the analysis of the effect size in the multi-factor model. The resulting pricing model explains the greater part of the size effect. To explain the size effect more clearly, Jagannathan and Wang (1996) included, in addition to the market wealth index, four other factors. The first, UTS_t , is the monthly return of the spread between long-term treasury bonds of the US government and short-term bonds (Treasury bill.) The second, UPR_t , is the return of the spread between long-term papers of US companies and the long-term US government papers as well. The third, MP_t , is the rate of

growth of industrial production monthly in the United States; and the fourth, UI_t corresponds to the variations in the inflation rate, according to the following models:

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$$E[R_{it}] = c_0 + c_{vw} \beta_{vw}^{vw} + c_{UTS} \beta_{UTS}^{UTS} + c_{MP} \beta_{MP}^{MP} + c_{UPR} \beta_{UPR}^{UPR} + c_{UI} \beta_{UI}^{UI} + c_{logit} \text{logit}_i \quad (4.4)$$

$$E[R_{it}] = c_0 + c_{vw} \beta_{vw}^{vw} + c_{prem} \beta_{prem}^{prem} + c_{labor} \beta_{labor}^{labor} + c_{UTS} \beta_{UTS}^{UTS} + c_{MP} \beta_{MP}^{MP} + c_{UPR} \beta_{UPR}^{UPR} + c_{UI} \beta_{UI}^{UI} + \text{Logit}_i \quad (4.5)$$

Where all betas are calculated in the same way, as in β^{vw} . Fama and French (1993) examined the relationship of beta with market size and for this they introduced two variables, Small Minus Big (SMB_t) and High Minus Low (HML_t) with the objective of capturing risks related to market size and, thereby, better explaining the cross-sectional variation of expected return.

Although Berk (1995) shows that the log of market size should be correlated with expected cross-sectional returns, such fact does not necessarily imply that this correlation can be captured by the model.

Therefore, Jagannathan and Wang (1996) examine whether the two factors, which Fama and French (1993) identify, are associated with the return on human capital and the instability of the beta of their PSI model. With this objective, they consider the following models:

$$E[R_{it}] = c_0 + c_{vw} \beta_{vw}^{vw} + c_{SMB} \beta_{SMB}^{SMB} + c_{HML} \text{logit}_i \quad (4.6)$$

$$E[R_{it}] = c_0 + c_{vw} \beta_{vw}^{vw} + c_{prem} \beta_{prem}^{prem} + c_{labor} \beta_{labor}^{labor} + c_{SMB} \beta_{SMB}^{SMB} + c_{HML} \text{logit}_i \quad (4.7)$$

Where all betas are calculated in the same way, as in β^{vw} .

Starting from these assumptions, it is verified that the empirical support for the specifications of the conditional CAPM is relatively strong. When betas and expected returns take into account long periods of time and assume that the CAPM is satisfactory period by period, the size effect and the statistical rejections of the model specifications become very weak.

When the proxy for the return on human capital is also included in the measure of the aggregate wealth return, the model pricing errors are not significant at conventional level, furthermore, the size effect shows no statistical significance.

VI. CONCLUSIONS AND RECOMMENDATIONS

Although the conditional model performs substantially better than the static model, caution is recommended in interpreting the results of the conditional CAPM for the following reasons: 1) The presented model of time-varying betas is still somewhat simple. In the dynamic world, investors can hedge against the variety of risks that do not exist in the static economy; in this sense, the NN can mitigate this factor 2) The analysis of the conditional CAPM is fundamental, as the specification of the unconditional CAPM can lead to false conclusions regarding the validity of the model. Some papers show that the conditional logit CAPM is not valid when we are analyzing very long time periods; 3) The number of deterministic events that occur with monthly and annual frequency.

It is expected that these events influence the behavior of asset pricing models in those periods. For events that are outside the scope of asset pricing models similar to the CAPM, one of the strategies should be to study the performance of models using annual data with the application of neural networks for a long period of time as in, Jagannathan and Wang (1992), and Kothari, Shanken, and Sloan (1995).

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